

MATH 347 HW 7

due October 30, at the beginning of class

HOMEWORK GUIDLINES

Obviously, your solutions need to be complete and correct, but to receive full credit your write-up should also satisfy the following:

- All the important logical steps in the proof should be present and fully explained.
- All assumptions should be clearly identified.
- Your solutions should be clear and concise. If a sentence does not further the reader's understanding of the solution then it has no place in your write up.
- Use full and grammatically correct English sentences. Mathematical symbols should be used only to render complex mathematical relationships into a readable form.

Moreover, in order to obtain full credit for the homework, you must write down, in the very least, an attempt at a solution for each problem.

PROBLEMS

Do the following problems from your book: 13.8, 13.9, 13.20, 13.21, 13.25. Also answer the following:

- (1) Let $\{a_n\}$ be a sequence of real numbers. Show that the sequence converges to $a \in \mathbb{R}$ if and only if

$$\lim_{n \rightarrow \infty} |a_n - a| = 0.$$

- (2) Let $\{a_n\}$ be a sequence of real numbers. Show that if

$$\lim_{n \rightarrow \infty} a_n = a,$$

then the set $\{a_n \mid n \in \mathbb{N}\}$ is bounded above and below.

- (3) Suppose X_n is a countably infinite set for each $n \in \mathbb{N}$. Show that the union

$$X = \bigcup_{n \in \mathbb{N}} X_n$$

is a countably infinite set.